Supplementary Materials for
Are robust standard errors the best approach for interval estimation with non-normal data in structural equation modeling?

Overview

The supplementary materials here contain additional results for Studies 1-3.

Study 1

Table 1 presents a count of the number of cases that had valid results for overall model fit in each condition - based on either normal theory likelihood (ML), or that with robust corrections (MLM). Cases that did not have valid results included those that did not converge to a solution, or those under which a valid test of model fit and *p*-value could not be computed. Table 2 presents the number of replications that had an improper estimate, operationalized as a negative variance estimate (Heywood case) or covariance matrix among latent variables that was not positive definite. Table 3 presents overall model rejection rates based only on valid replications. Tables 4 through 9 present empirical standard deviations and bias for the parameters of interest in Study 1 (i.e., first factor loading, first error variance, factor covariance). Figure 1 presents the proportion of cases (rounded to 2 decimal places) under each cell of the design with a valid CI. Figure 2 presents median CI width.

Table 1: Number of cases under each condition in Study 1 where overall model estimation succeeded

N	Condition	ML	MLM
100	Normal	1000	1000
200	Normal	1000	1000
500	Normal	1000	1000
100	VM Moderate	999	999
200	VM Moderate	1000	1000
500	VM Moderate	1000	1000
100	VM Severe	1000	1000
200	VM Severe	1000	1000
500	VM Severe	1000	1000
100	CN Severe	984	979
200	CN Severe	999	999
500	CN Severe	1000	1000
100	CN Moderate	999	998
200	CN Moderate	1000	1000
500	CN Moderate	1000	1000
100	Copula Moderate	1000	1000
200	Copula Moderate	1000	1000
500	Copula Moderate	1000	1000
100	Copula Severe	999	999
200	Copula Severe	1000	1000
500	Copula Severe	1000	1000

Note. VM = Vale and Maurelli method; CN = Contaminated normal; Copula = Multivariate Gumbel Copula; ML = Maximum likelihood estimation; MLM = Maximum likelihood estimation with robust corrections. Note that the number of cases for ML and MLM differs slightly in some cases due to the overall test statistic not being computable for MLM.

Table 2: Number of solutions with one or more improper parameter estimates

N	Condition	Improper parameter estimates
100	Normal Normal	0
200	Normal Normal	0
500	Normal Normal	0
100	VM moderate	3
200	VM moderate	0
500	VM moderate	0
100	VM severe	1
200	VM severe	0
500	VM severe	0
100	CN moderate	11
200	CN moderate	0
500	CN moderate	0
100	CN severe	52
200	CN severe	3
500	CN severe	0
100	Copula moderate	0
200	Copula moderate	0
500	Copula moderate	0
100	Copula severe	10
200	Copula severe	0
500	Copula severe	0

Note. VM = Vale and Maurelli method; CN = Contaminated normal; Copula = Multivariate Gumbel Copula.

Table 3: Model rejection rates under each condition in Study 1

N	Condition	ML	MLM
100	Normal	0.06	0.08
200	Normal	0.06	0.06
500	Normal	0.06	0.06
100	VM Moderate	0.21	0.09
200	VM Moderate	0.19	0.07
500	VM Moderate	0.21	0.06
100	VM Severe	0.19	0.09
200	VM Severe	0.19	0.07
500	VM Severe	0.24	0.06
100	CN Moderate	0.58	0.08
200	CN Moderate	0.59	0.06
500	CN Moderate	0.59	0.05
100	CN Severe	0.92	0.07
200	CN Severe	0.90	0.07
500	CN Severe	0.91	0.06
100	Copula Moderate	0.20	0.06
200	Copula Moderate	0.19	0.06
500	Copula Moderate	0.18	0.06
100	Copula Severe	0.39	0.08
200	Copula Severe	0.37	0.06
500	Copula Severe	0.39	0.06

Note. VM = Vale and Maurelli method; CN = Contaminated normal; Copula = Multivariate Gumbel Copula; ML = Maximum likelihood estimation using the normal theory chi-square test of overall fit; MLM = Maximum likelihood estimation with robust corrections, using a scaled chi-square test of overall fit.

Table 4: Empirical standard deviations for first factor loading, Study 1

N	Condition	Empirical SD
100	Normal	0.18
200	Normal	0.11
500	Normal	0.07
100	VM moderate	0.27
200	VM moderate	0.16
500	VM moderate	0.10
100	VM severe	0.31
200	VM severe	0.18
500	VM severe	0.11
100	CN moderate	0.39
200	CN moderate	0.16
500	CN moderate	0.10
100	CN severe	0.88
200	CN severe	0.25
500	CN severe	0.12
100	Copula moderate	0.23
200	Copula moderate	0.12
500	Copula moderate	0.07
100	Copula severe	0.24
200	Copula severe	0.15
500	Copula severe	0.09

Note. VM = Vale and Maurelli method; CN = Contaminated normal; Copula = Multivariate Gumbel Copula.

Table 5: Raw bias for first factor loading, Study 1

N	Condition	Bias
100	Normal	0.02
200	Normal	0.01
500	Normal	0.00
100	VM moderate	0.04
200	VM moderate	0.02
500	VM moderate	0.01
100	VM severe	0.05
200	VM severe	0.02
500	VM severe	0.01
100	CN severe	0.12
200	CN severe	0.04
500	CN severe	0.01
100	CN moderate	0.07
200	CN moderate	0.01
500	CN moderate	0.01
100	Copula moderate	0.04
200	Copula moderate	0.01
500	Copula moderate	0.00
100	Copula severe	0.04
200	Copula severe	0.02
500	Copula severe	0.01

Note. VM = Vale and Maurelli method; CN = Contaminated normal; Copula = Multivariate Gumbel Copula.

Table 6: Empirical standard deviations for first error variance, Study 1

N	Condition	Fixed Empirical SD
100	Normal	0.13
200	Normal	0.09
500	Normal	0.06
100	VM moderate	0.22
200	VM moderate	0.15
500	VM moderate	0.10
100	VM severe	0.28
200	VM severe	0.20
500	VM severe	0.13
100	CN moderate	0.18
200	CN moderate	0.11
500	CN moderate	0.08
100	CN severe	0.21
200	CN severe	0.14
500	CN severe	0.08
100	Copula moderate	0.14
200	Copula moderate	0.10
500	Copula moderate	0.07
100	Copula severe	0.18
200	Copula severe	0.13
500	Copula severe	0.08

Note. VM = Vale and Maurelli method; CN = Contaminated normal; Copula = Multivariate Gumbel Copula.

Table 7: Raw bias for first first error variance, Study 1

N	Condition	Bias
100	Normal	-0.01
200	Normal	-0.01
500	Normal	-0.01
100	VM moderate	-0.02
200	VM moderate	-0.02
500	VM moderate	0.00
100	VM severe	-0.04
200	VM severe	-0.01
500	VM severe	-0.01
100	CN moderate	-0.03
200	CN moderate	-0.01
500	CN moderate	-0.00
100	CN severe	-0.02
200	CN severe	-0.03
500	CN severe	0.00
100	Copula moderate	-0.02
200	Copula moderate	0.00
500	Copula moderate	0.00
100	Copula severe	-0.02
200	Copula severe	-0.01
500	Copula severe	0.00

Note. VM = Vale and Maurelli method; CN = Contaminated normal; Copula = Multivariate Gumbel Copula.

Table 8: Empirical standard deviations for factor covariance, Study 1

N	Condition	Empirical SD
100	Normal	0.18
200	Normal	0.12
500	Normal	0.07
100	VM moderate	0.21
200	VM moderate	0.14
500	VM moderate	0.08
100	VM severe	0.21
200	VM severe	0.14
500	VM severe	0.08
100	CN moderate	0.24
200	CN moderate	0.16
500	CN moderate	0.10
100	CN severe	0.30
200	CN severe	0.20
500	CN severe	0.12
100	Copula moderate	0.18
200	Copula moderate	0.13
500	Copula moderate	0.08
100	Copula severe	0.20
200	Copula severe	0.13
500	Copula severe	0.08

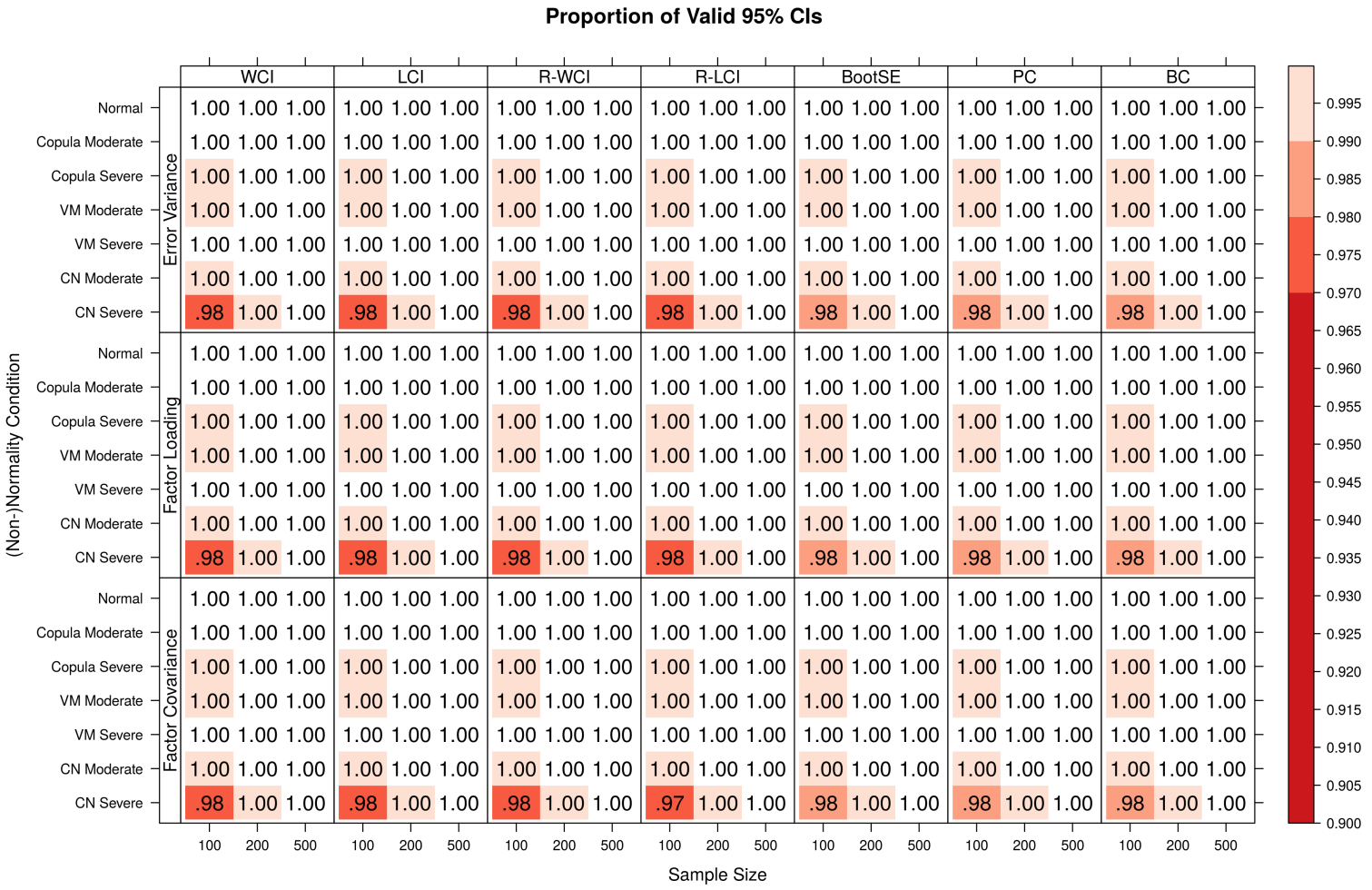
Note. VM = Vale and Maurelli method; CN = Contaminated normal; Copula = Multivariate Gumbel Copula.

Table 9: Raw bias for factor covariance, Study 1

N	Condition	Bias
100	Normal	0.00
200	Normal	0.01
500	Normal	0.00
100	VM moderate	-0.01
200	VM moderate	-0.01
500	VM moderate	0.00
100	VM severe	-0.01
200	VM severe	0.00
500	VM severe	-0.01
100	CN moderate	0.00
200	CN moderate	0.00
500	CN moderate	0.00
100	CN severe	0.00
200	CN severe	0.00
500	CN severe	0.00
100	Copula moderate	0.00
200	Copula moderate	0.00
500	Copula moderate	0.00
100	Copula severe	0.03
200	Copula severe	0.02
500	Copula severe	0.01

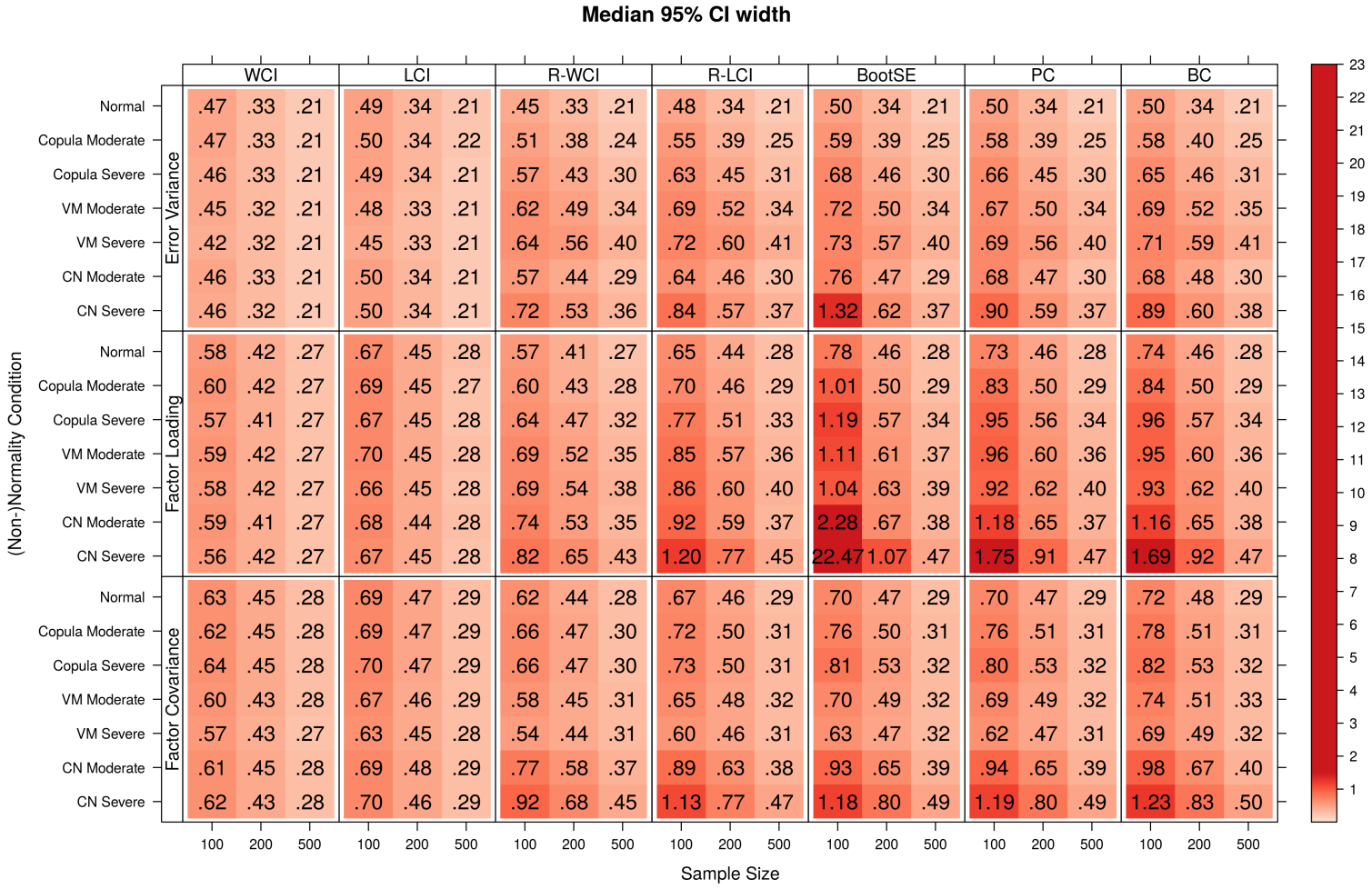
Note. VM = Vale and Maurelli method; CN = Contaminated normal; Copula = Multivariate Gumbel Copula.

Figure 1: Proportion of valid confidence intevals for Study 1



Note. VM = Vale and Maurelli method; CN = Contaminated Normal; Copula = Multivariate Gumbel Copula; WCI = Wald-based confidence interval; LCI = Likelihood-based confidence interval; R-WCI = Robust wald-based confidence intervals; R-LCI = Robust likelihood-based confidence intervals; BootSE = Bootstrap standard errors; PC = Percentile bootstrap; BC = Bias-corrected bootstrap.

Figure 2: Median interval width for confidence intervals in Study 1



Note. VM = Vale and Maurelli method; CN = Contaminated Normal; Copula = Multivariate Gumbel Copula; WCI = Wald-based confidence interval; LCI = Likelihood-based confidence interval; R-WCI = Robust wald-based confidence intervals; R-LCI = Robust likelihood-based confidence intervals; BootSE = Bootstrap standard errors; PC = Percentile bootstrap; BC = Bias-corrected bootstrap.

Study 2

Table 10 presents a count of the number of cases that had valid results for overall model fit in each condition - based on either normal theory likelihood (ML), or that with robust corrections (MLM). Table 12 presents overall model rejection rates based only on valid replications.

Figure 3 presents the proportion of cases (rounded to 2 decimal places) under each cell of the design with a valid CI. Figure 4 presents median CI width. Figure 5 presents relative efficiency. Note that the robust likehood-based approach (R-LCI) is used as the benchmark for relative efficiency such that the median width for any given approach in any cell is divided by the median width for R-LCI in the same cell. Thus, values greater than 1 indicate larger intervals and are less efficient than R-LCI. Figure 6 presents the proportion of intervals with widths greater than 1.

Table 10: Number of cases under each condition in Study 2 where overall model estimation succeeded

N	Condition	ML	MLM
75	Normal	1000	1000
125	Normal	1000	1000
300	Normal	1000	1000
1000	Normal	1000	1000
75	VM	1000	1000
125	VM	1000	1000
300	VM	1000	1000
1000	VM	1000	1000
75	CN	1000	1000
125	CN	1000	1000
300	CN	1000	1000
1000	CN	1000	1000
75	NNDP1	1000	999
125	NNDP1	999	999
300	NNDP1	1000	1000
1000	NNDP1	1000	1000
75	NNDP2	1000	1000
125	NNDP2	1000	1000
300	NNDP2	1000	1000
1000	NNDP2	1000	1000

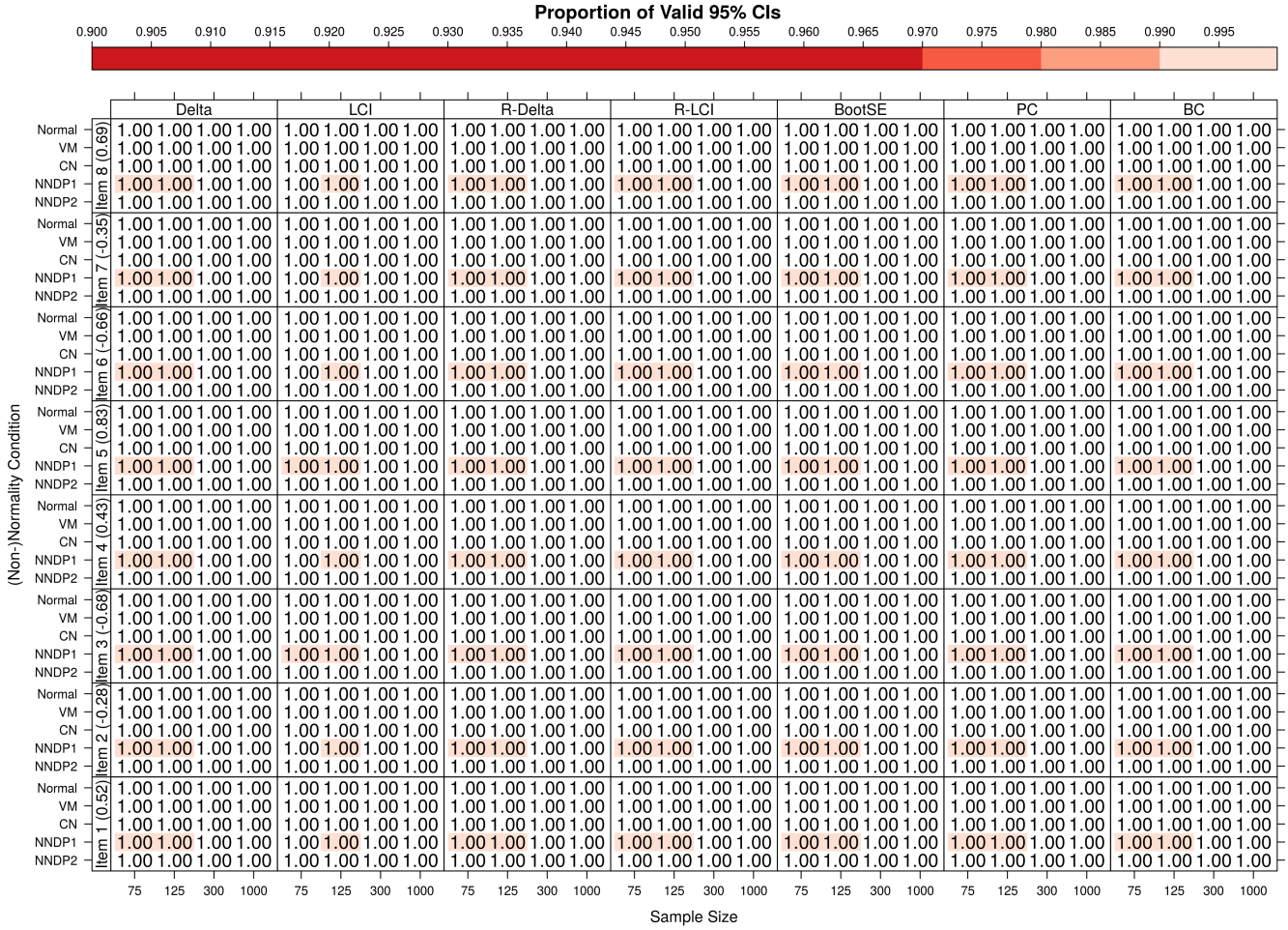
Note. VM = Vale and Maurelli method; CN = Contaminated normal; NNDP1 = Non-normal with dependency condition 1; NNDP2 = Non-normal with dependency condition 2; ML = Maximum likelihood estimation; MLM = Maximum likelihood estimation with robust corrections.

Table 11: Model rejection rates under each condition in Study 2

N	NNType	ML	MLM
75	Normal	0.08	0.09
125	Normal	0.07	0.08
300	Normal	0.06	0.07
1000	Normal	0.05	0.06
75	VM	0.14	0.14
125	VM	0.11	0.10
300	VM	0.13	0.09
1000	VM	0.11	0.07
75	CN	0.81	0.10
125	CN	0.81	0.08
300	CN	0.79	0.07
1000	CN	0.81	0.06
75	NNDP1	0.57	0.11
125	NNDP1	0.61	0.07
300	NNDP1	0.66	0.05
1000	NNDP1	0.81	0.06
75	NNDP2	0.42	0.13
125	NNDP2	0.51	0.11
300	NNDP2	0.51	0.06
1000	NNDP2	0.63	0.05

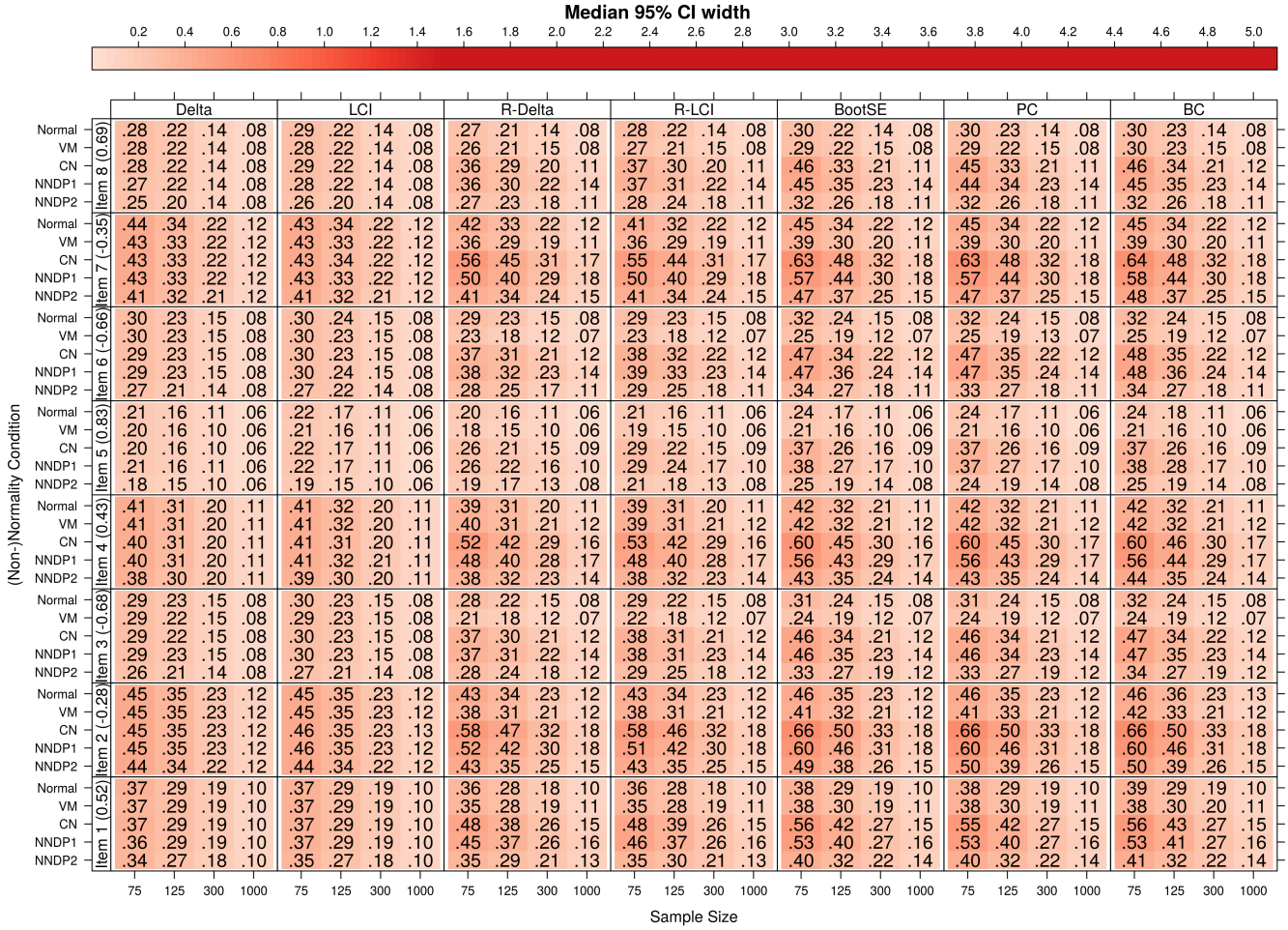
Note. VM = Vale and Maurelli method; CN = Contaminated normal; NNDP1 = Non-normal with dependency condition 1; NNDP2 = Non-normal with dependency condition 2; ML = Maximum likelihood estimation; MLM = Maximum likelihood estimation with robust corrections.

Figure 3: Proportion of valid confidence intervals, Study 2



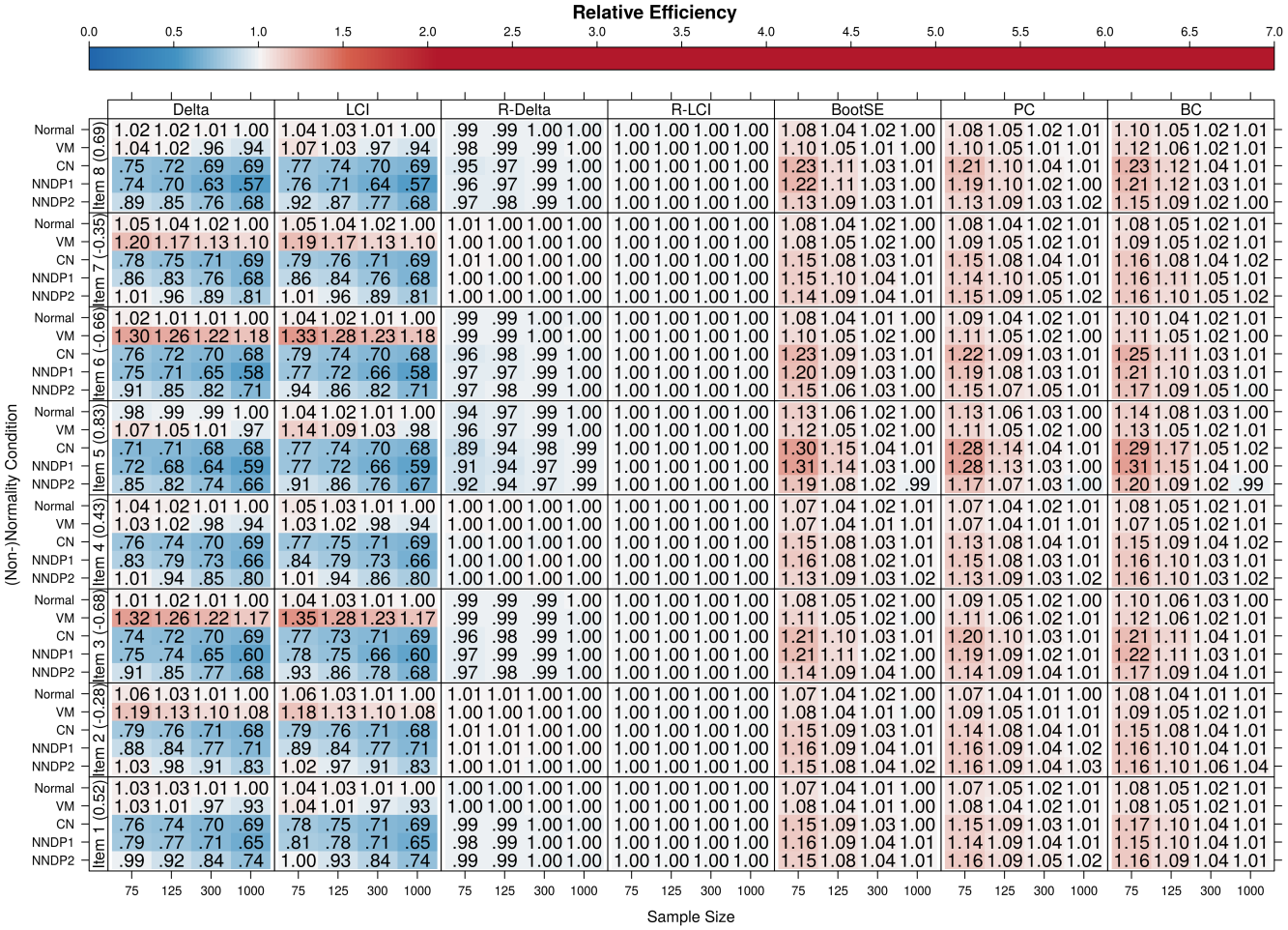
Note. VM = Vale and Maurelli method; CN = Contaminated Normal; NNDP1 = Non-normal with dependency 1; NNDP2 = Non-normal with dependency 2; WCI = Wald-based confidence interval; LCI = Likelihood-based confidence interval; R-WCI = Robust wald-based confidence intervals; R-LCI = Robust likelihood-based confidence intervals; BootSE = Bootstrap standard errors; PC = Percentile bootstrap; BC = Bias-corrected bootstrap.

Figure 4: Median interval width for confidence intervals, Study 2



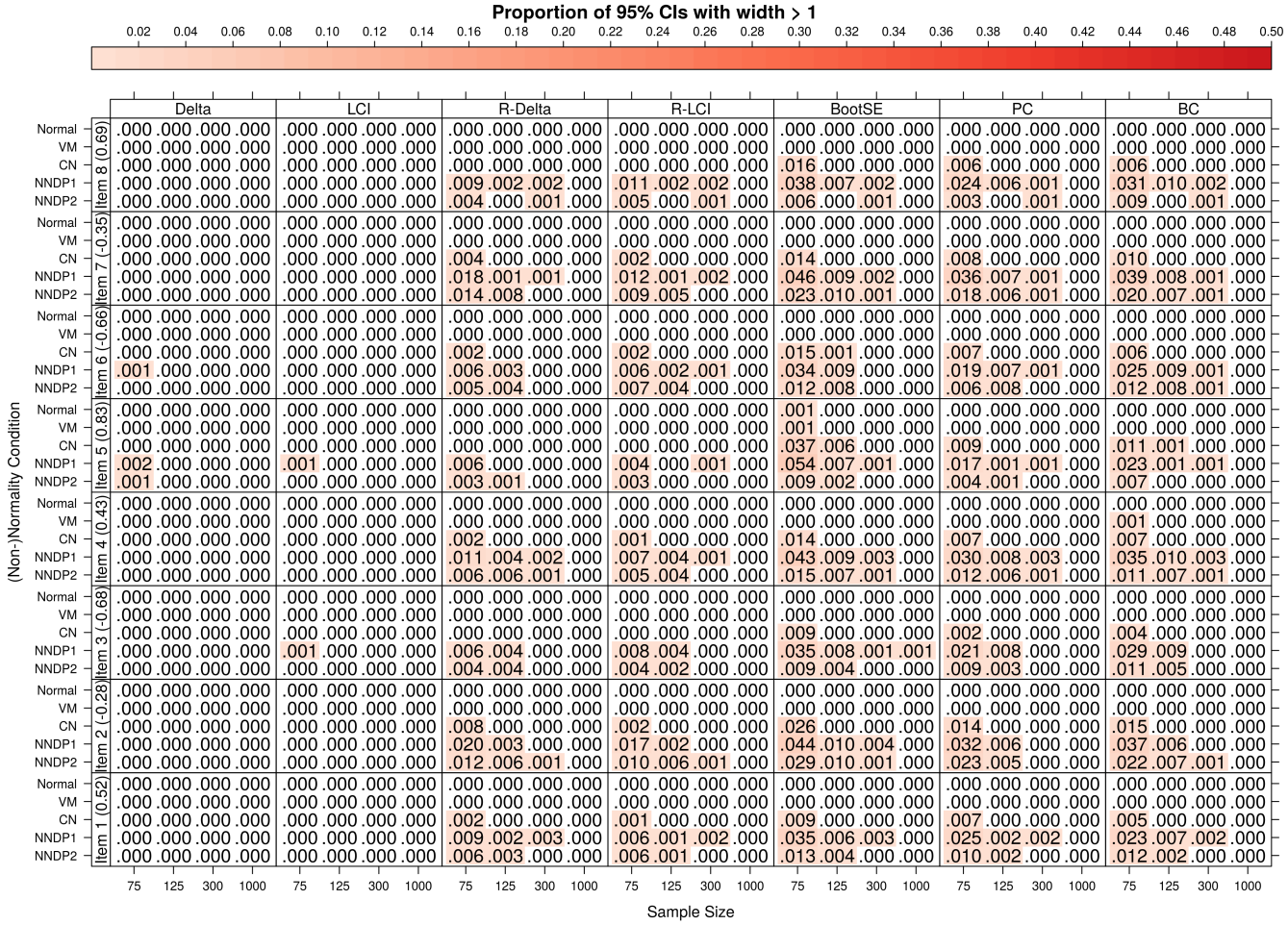
Note. VM = Vale and Maurelli method; CN = Contaminated Normal; NNDP1 = Non-normal with dependency 1; NNDP2 = Non-normal with dependency 2; WCI = Wald-based confidence interval; LCI = Likelihood-based confidence interval; R-WCI = Robust wald-based confidence intervals; R-LCI = Robust likelihood-based confidence intervals; BootSE = Bootstrap standard errors; PC = Percentile bootstrap; BC = Bias-corrected bootstrap.

Figure 5: Relative efficiency of median confidence intervals, Study 2



Note. VM = Vale and Maurelli method; CN = Contaminated Normal; NNDP1 = Non-normal with dependency 1; NNDP2 = Non-normal with dependency 2; WCI = Wald-based confidence interval; LCI = Likelihood-based confidence interval; R-WCI = Robust wald-based confidence intervals; R-LCI = Robust likelihood-based confidence intervals; BootSE = Bootstrap standard errors; PC = Percentile bootstrap; BC = Bias-corrected bootstrap.

Figure 6: Proportion of confidence intervals with widths > 1, Study 2



Note. VM = Vale and Maurelli method; CN = Contaminated Normal; NNDP1 = Non-normal with dependency 1; NNDP2 = Non-normal with dependency 2; WCI = Wald-based confidence interval; LCI = Likelihood-based confidence interval; R-WCI = Robust wald-based confidence intervals; R-LCI = Robust likelihood-based confidence intervals; BootSE = Bootstrap standard errors; PC = Percentile bootstrap; BC = Bias-corrected bootstrap.

Study 3

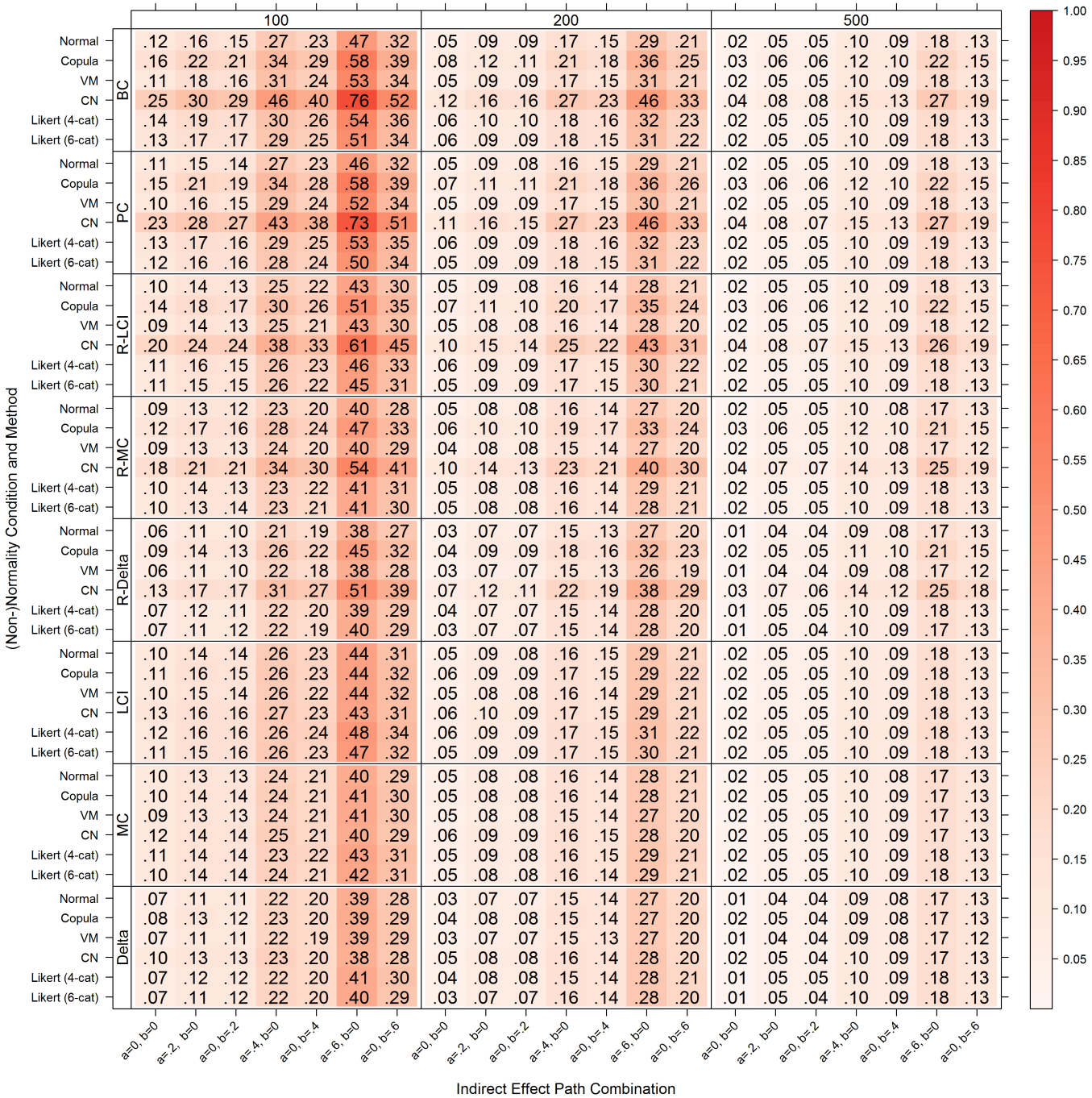
Note that there were no overall model estimation problems in Study 3. Overall model rejection rates in Table 12 are collapsed across across indirect effect path combinations. Operationalized in the same way as in Study 1, we also present median interval width (Figures 7 and 8), relative efficiency (Figures 9 and 10), and the proportion of cases with outlying interval widths (Figures 11 and 12) for each cell of the design.

Table 12: Model rejection rates under each data generation condition in Study 3

N	Condition	ML	MLM
100	Normal	0.09	0.12
200	Normal	0.07	0.08
500	Normal	0.05	0.06
100	VM	0.42	0.15
200	VM	0.47	0.10
500	VM	0.55	0.07
100	CN	0.99	0.14
200	CN	0.99	0.08
500	CN	0.99	0.06
100	Copula	0.67	0.12
200	Copula	0.67	0.08
500	Copula	0.67	0.06
100	Likert (4-cat)	0.14	0.12
200	Likert (4-cat)	0.11	0.08
500	Likert (4-cat)	0.09	0.06
100	Likert (6-cat)	0.13	0.12
200	Likert (6-cat)	0.10	0.08
500	Likert (6-cat)	0.09	0.06

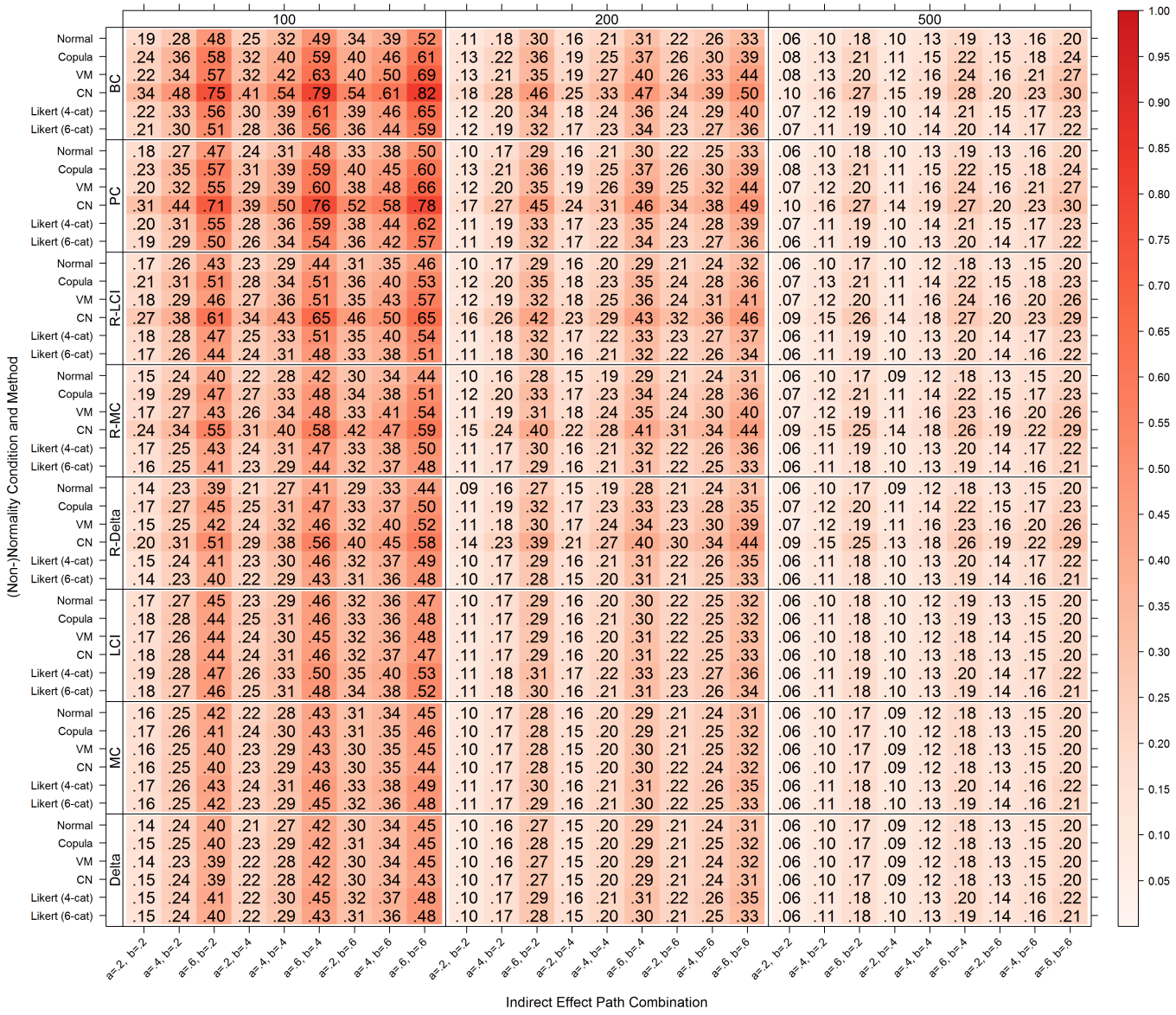
Note. VM = Vale and Maurelli method; CN = Contaminated normal; Copula = Multivariate Gumbel Copula; Likert (4-cat) = descretized data into 4 categories; Likert (6-cat) = descretized data into 6 categories; ML = Maximum likelihood estimation using the normal theory chi-square test of overall fit; MLM = Maximum likelihood estimation with robust corrections, using a scaled chi-square test of overall fit.

Figure 7: Median interval width for confidence intervals in Study 3, $ab = 0$



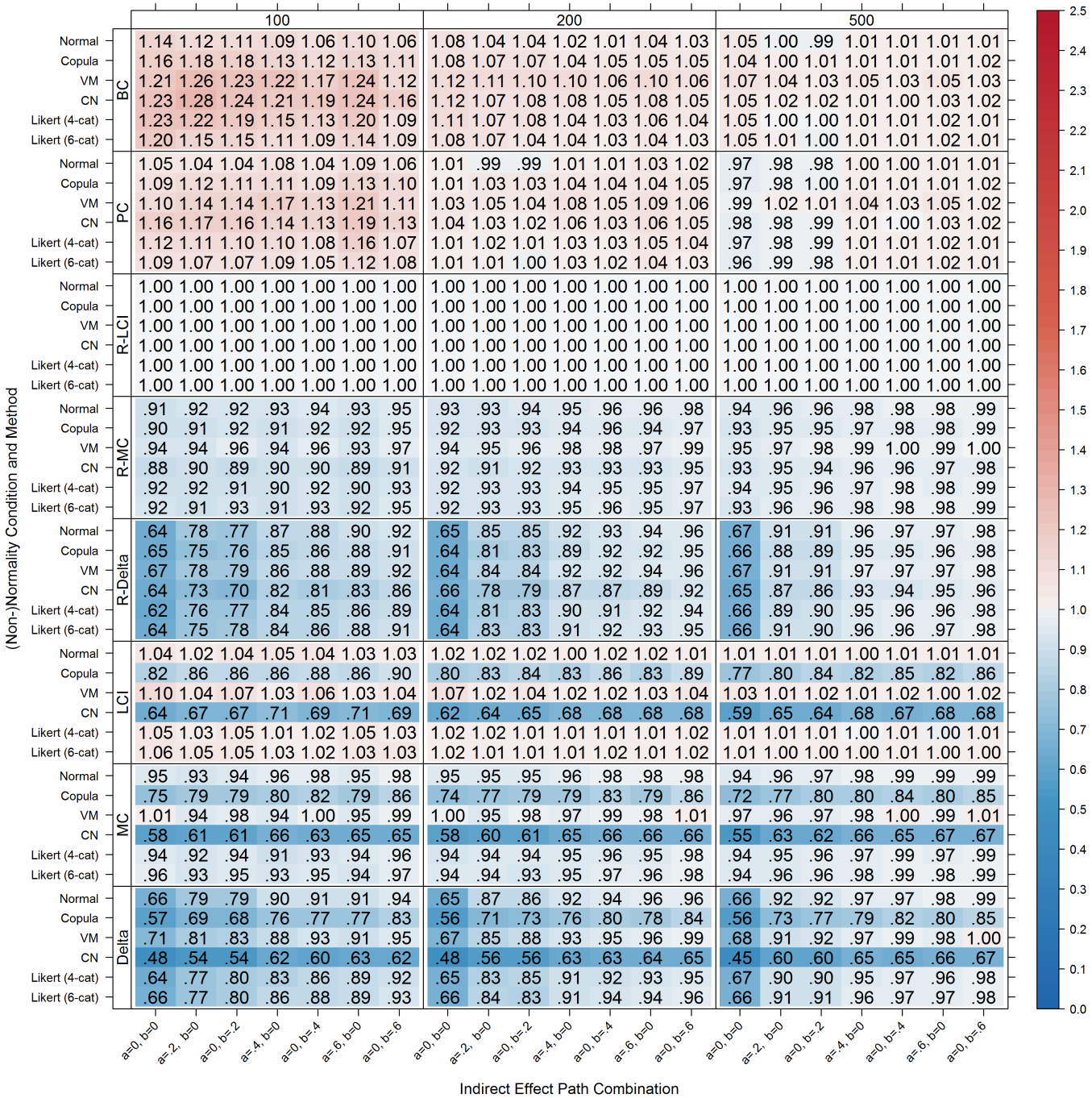
Note. VM = Vale and Maurelli method; CN = Contaminated Normal; Copula = Multivariate Gumbel Copula; Likert (4-cat) = descritized into 4-category data; Likert (6-cat) = descritized into 6-category data; R- = Robust (prefix added to methods); Delta = Delta method-based confidence interval; MC = Monte Carlo method-based confidence intervals; LCI = Likelihood-based confidence interval; PC = Percentile bootstrap; BC = Bias-corrected bootstrap.

Figure 8: Median interval width for confidence intervals in Study 3, $ab \neq 0$



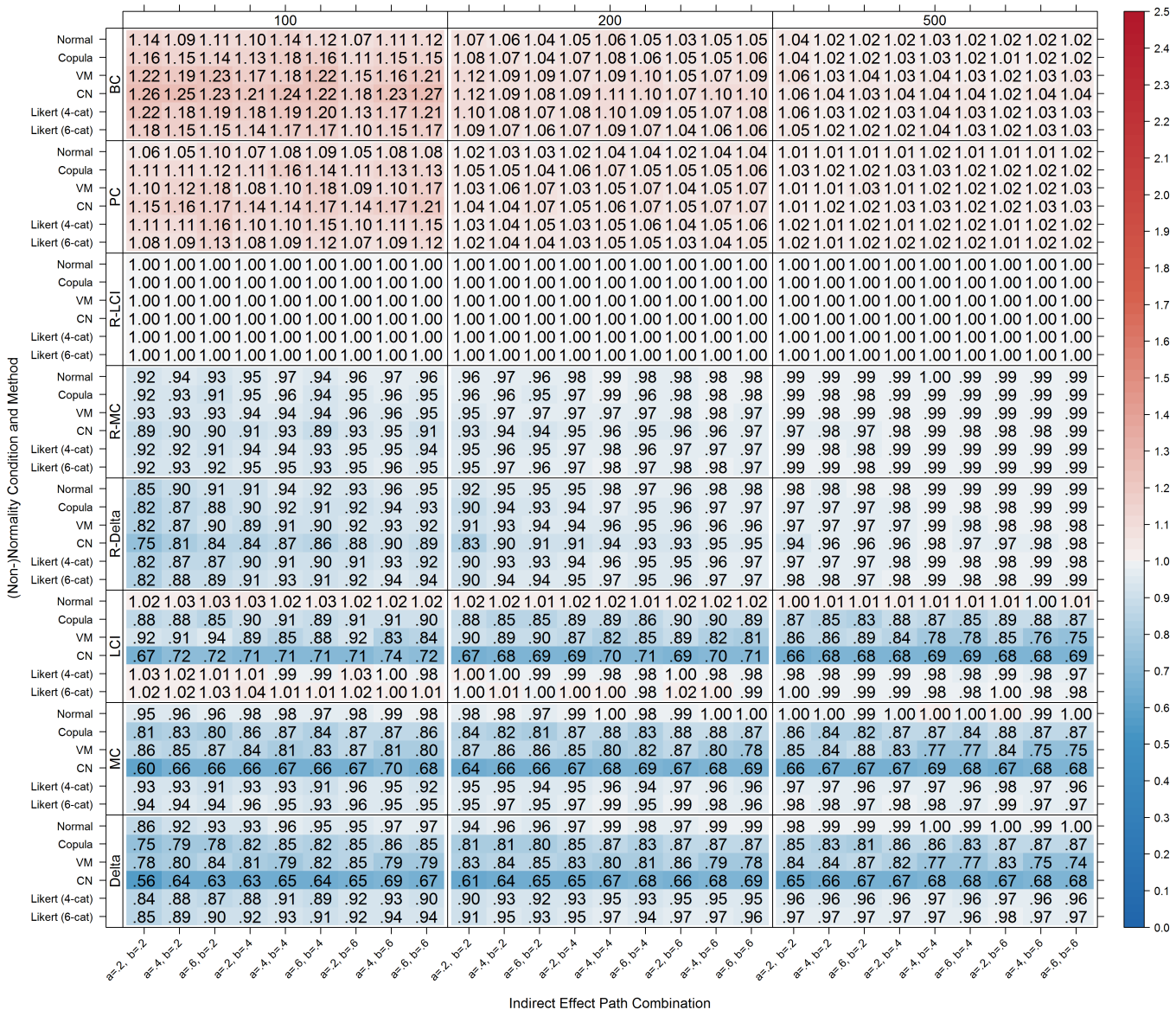
Note. VM = Vale and Maurelli method; CN = Contaminated Normal; Copula = Multivariate Gumbel Copula; Likert (4-cat) = descritized into 4-category data; Likert (6-cat) = descritized into 6-category data; R- = Robust (prefix added to methods); Delta = Delta method-based confidence interval; MC = Monte Carlo method-based confidence intervals; LCI = Likelihood-based confidence interval; PC = Percentile bootstrap; BC = Bias-corrected bootstrap.

Figure 9: Relative efficiency of median confidence intevals for Study 3, $ab = 0$



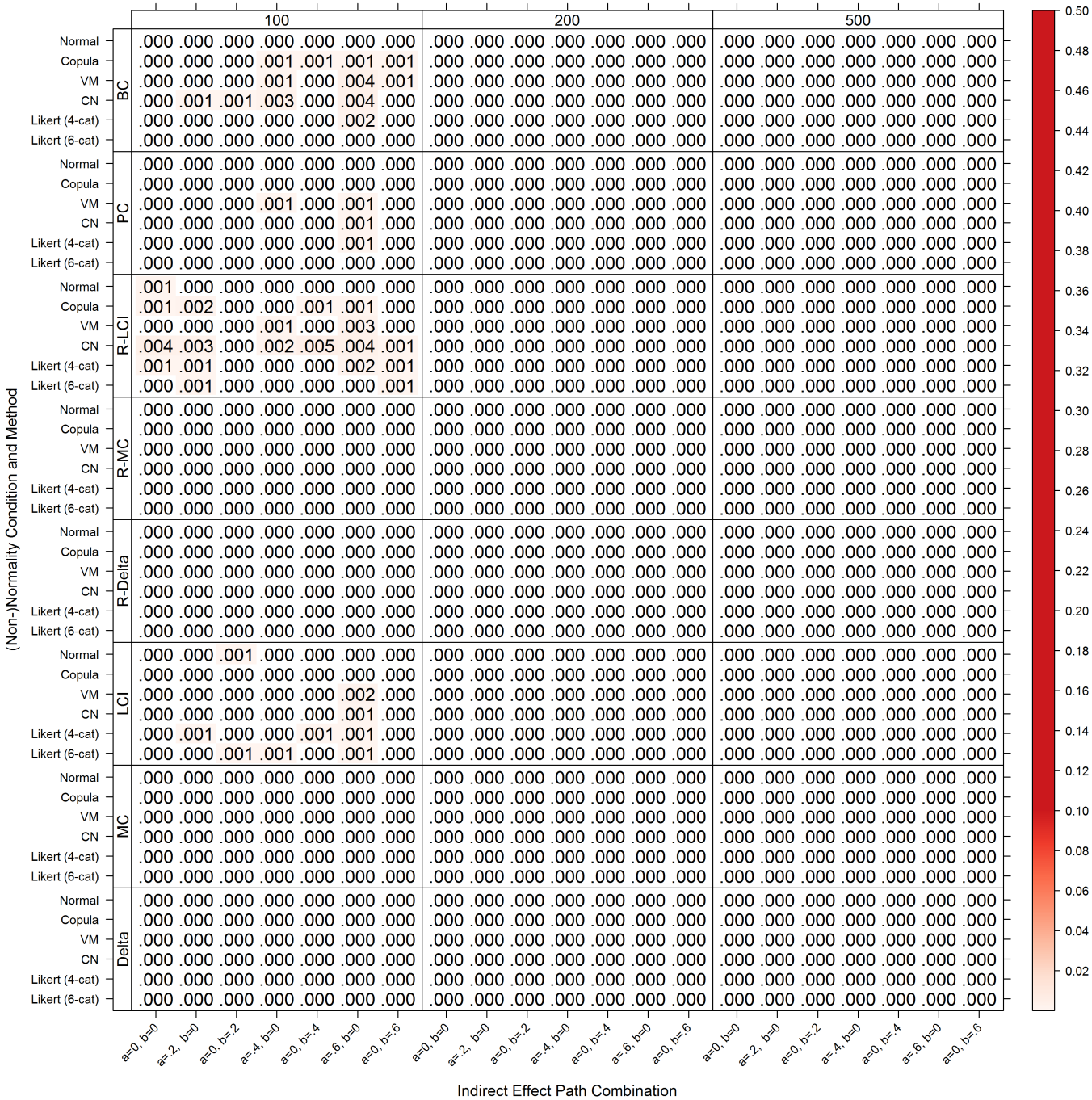
Note. VM = Vale and Maurelli method; CN = Contaminated Normal; Copula = Multivariate Gumbel Copula; Likert (4-cat) = descritized into 4-category data; Likert (6-cat) = descritized into 6-category data; R- = Robust (prefix added to methods); Delta = Delta method-based confidence interval; MC = Monte Carlo method-based confidence intervals; LCI = Likelihood-based confidence interval; PC = Percentile bootstrap; BC = Bias-corrected bootstrap.

Figure 10: Relative efficiency of median confidence intevals for Study 3, $ab \neq 0$



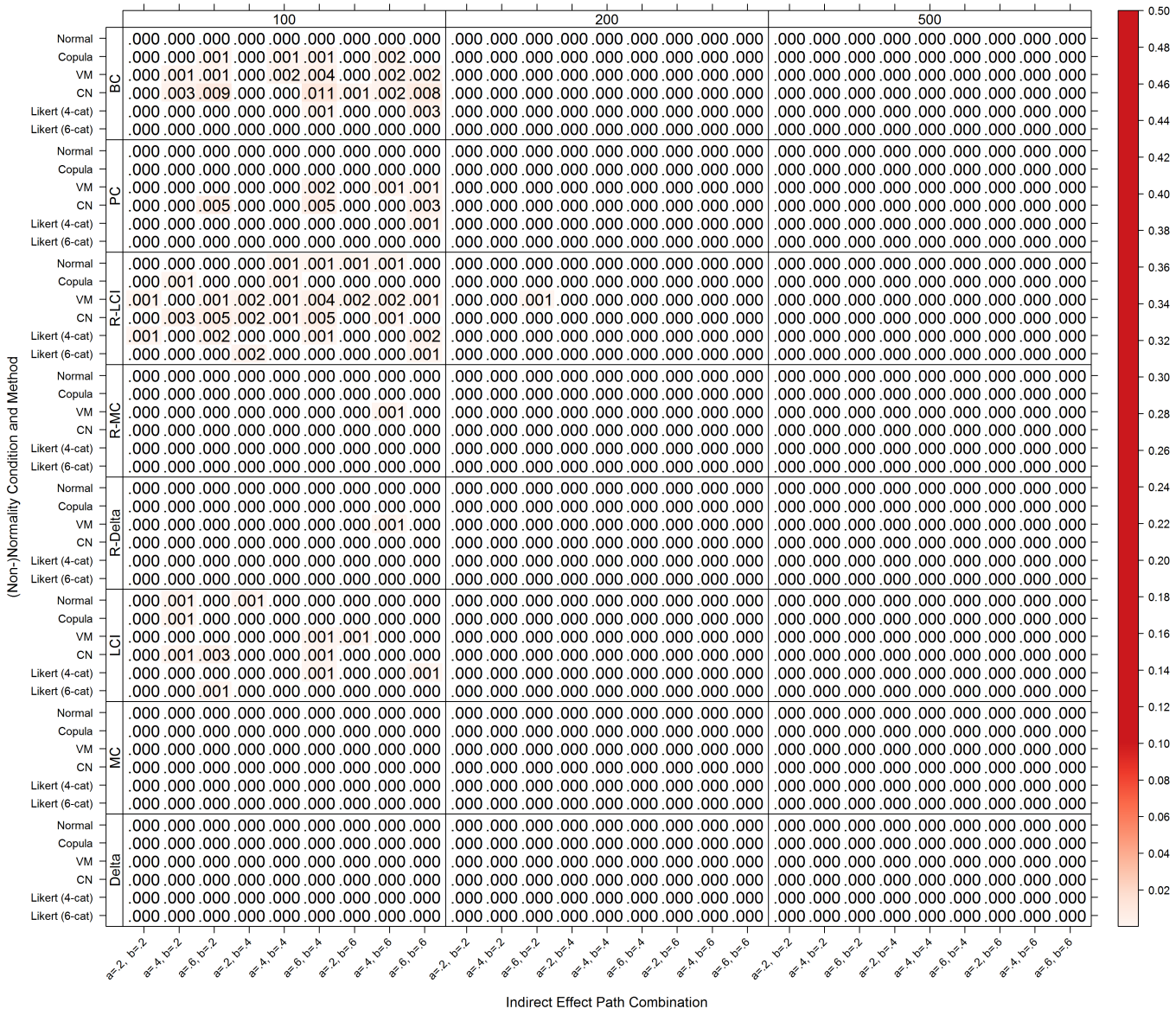
Note. VM = Vale and Maurelli method; CN = Contaminated Normal; Copula = Multivariate Gumbel Copula; Likert (4-cat) = descritized into 4-category data; Likert (6-cat) = descritized into 6-category data; R- = Robust (prefix added to methods); Delta = Delta method-based confidence interval; MC = Monte Carlo method-based confidence intervals; LCI = Likelihood-based confidence interval; PC = Percentile bootstrap; BC = Bias-corrected bootstrap.

Figure 11: Proportion of intervals wider than 5 for Study 3, $ab = 0$



Note. VM = Vale and Maurelli method; CN = Contaminated Normal; Copula = Multivariate Gumbel Copula; Likert (4-cat) = descritized into 4-category data; Likert (6-cat) = descritized into 6-category data; R- = Robust (prefix added to methods); Delta = Delta method-based confidence interval; MC = Monte Carlo method-based confidence intervals; LCI = Likelihood-based confidence interval; PC = Percentile bootstrap; BC = Bias-corrected bootstrap.

Figure 12: Proportion of intervals wider than 5 for Study 3, $ab \neq 0$



Note. VM = Vale and Maurelli method; CN = Contaminated Normal; Copula = Multivariate Gumbel Copula; Likert (4-cat) = descritized into 4-category data; Likert (6-cat) = descritized into 6-category data; R- = Robust (prefix added to methods); Delta = Delta method-based confidence interval; MC = Monte Carlo method-based confidence intervals; LCI = Likelihood-based confidence interval; PC = Percentile bootstrap; BC = Bias-corrected bootstrap.