Functional linear models for scalar responses

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- With functional responses and multivariate independent variables we could estimate the regression coefficient functions without necessarily needing to use roughness penalties.
- The same with functional responses, functional independent variables and the concurrent model.
- Now we look at a scalar response predicted by a functional independent variable, and discover that a roughness penalty or *regularization* is indispensable.

A model for total annual precipitation

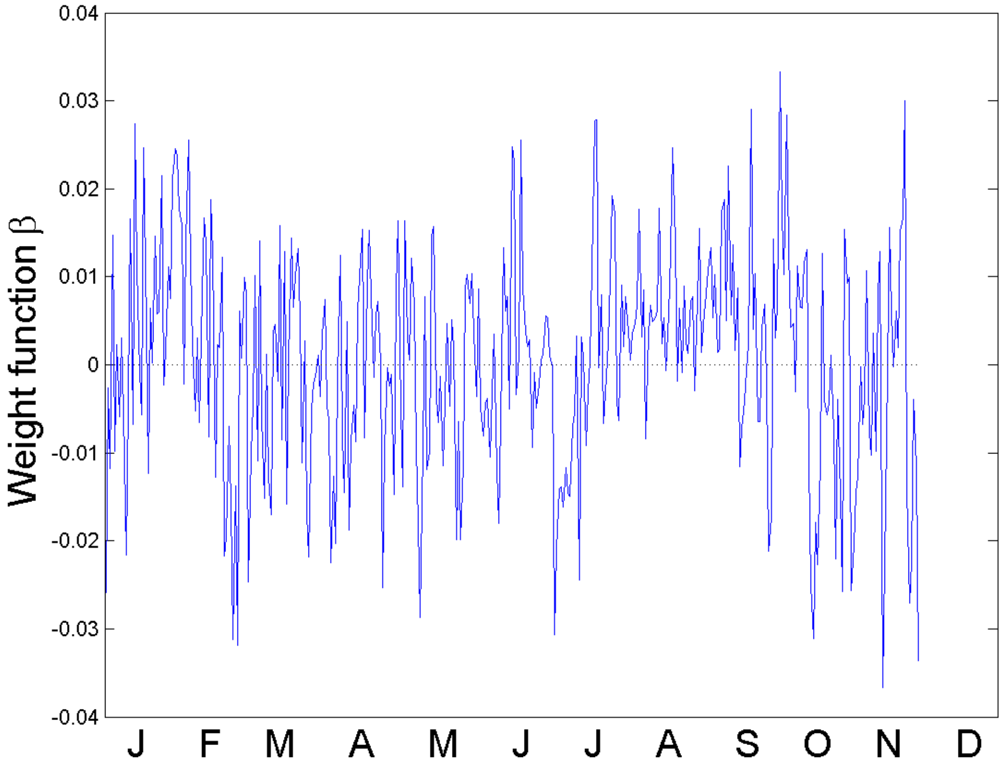
- Let $y_i = \texttt{LogPrec}_i$ be the logarithm of total annual precipitation at weather station i.
- Here is our model:

$$\texttt{LogPrec}_i = \alpha + \int_0^T \texttt{Temp}_i(s)\beta(s)\,ds + \epsilon_i\ .$$

- We can think of each value $\texttt{Temp}(s)$ as a separate scalar independent variable.
- If so, we have enough fitting power at our disposal to fit any number of responses, and certainly only 35 of them.

A bad idea

- If we use the discrete daily temperature averages, we have 365 plus 1 for constant α independent variables to fit 35 responses.
- Using the Moore-Penrose generalized inverse to keep us out of trouble, we get the following estimate of β.


0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
Weight function β
J F M A M J J A S O N D

Estimating $\beta(s)$ with a roughness penalty

- We could impose smoothness on $\beta(s)$ by expanding it in terms of a small number (< 35) of basis functions.
- Using a roughness penalty, however, gives us continuous control over smoothness and other advantages.
- Here is the penalized least squares criterion:

$$\text{PENSSE}_\lambda(\alpha, \beta) = \sum_{i=1}^{N} [y_i - \alpha - \int z_i(s)\beta(s)\, ds]^2 + \lambda \int [L\beta(s)]^2\, ds \,,$$

Choosing a roughness penalty

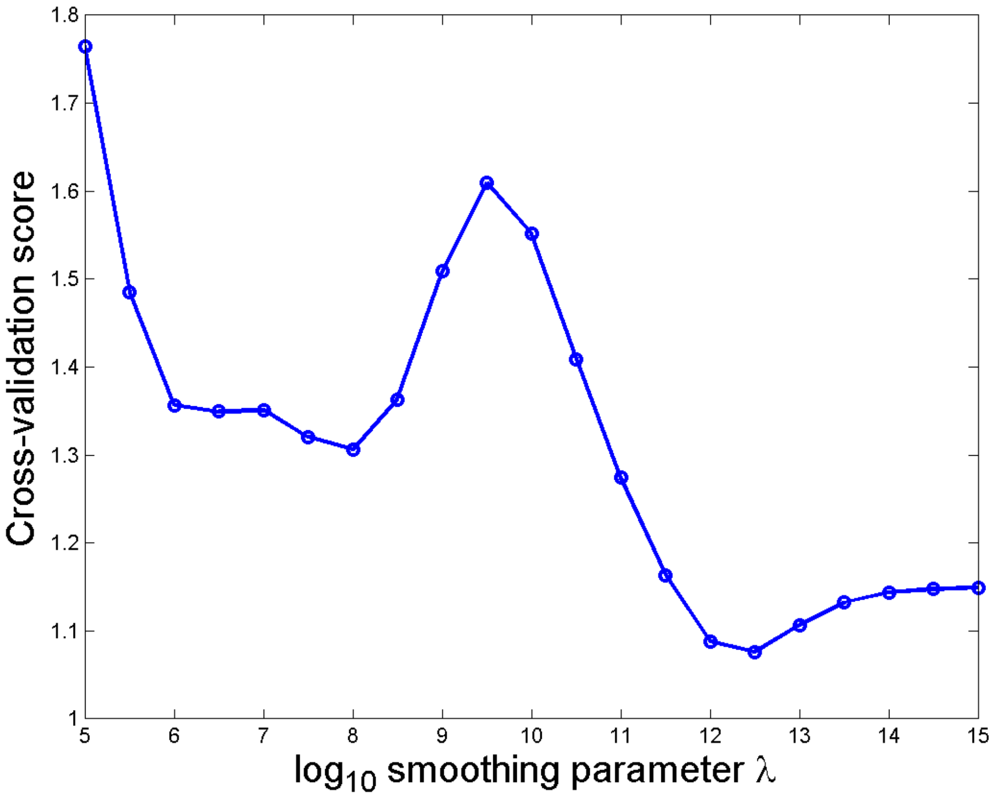
- Let's penalize *harmonic acceleration* because we want $\beta(s)$ to be periodic:

$$L\beta(s) = (\frac{2\pi}{365})^2 D\beta(s) + D^3\beta(s)$$

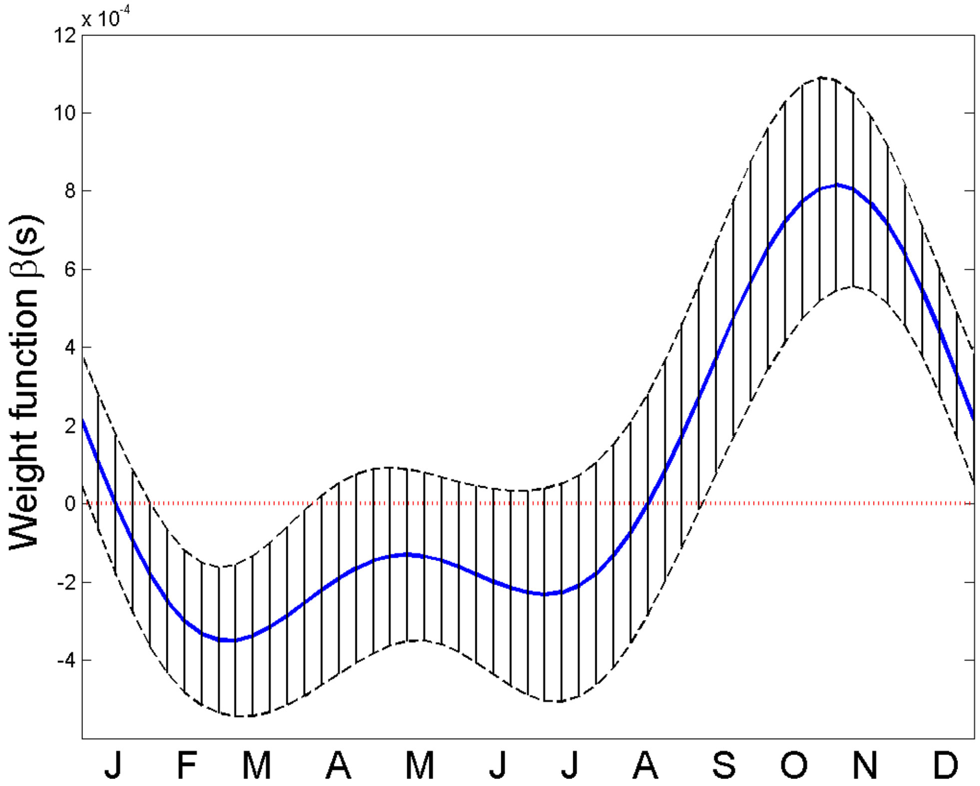
- We choose the smoothing parameter λ by minimizing the cross-validation criterion.
- Let $\alpha_\lambda^{(-i)}$ and $\beta_\lambda^{(-i)}$ be the estimates using all the responses except y_i.
- The criterion to be minimized is

$$\text{CV}(\lambda) = \sum_{i=1}^{N} [y_i - \alpha_\lambda^{(-i)} - \int z_i(s)\beta_\lambda^{(-i)}(s)\, ds]^2$$

A plot of CV(λ)



$\beta(s)$ **for** $\log_{10}\lambda = 12.5$



A plot of the fit