Functional responses, functional covariates and the full model

1. The full model for log precipitation

- We now want to predict the log precipitation profile $\texttt{LogPrec}_i(t)$ at time t from the entire temperature profile $\texttt{Temp}_i(s)$.
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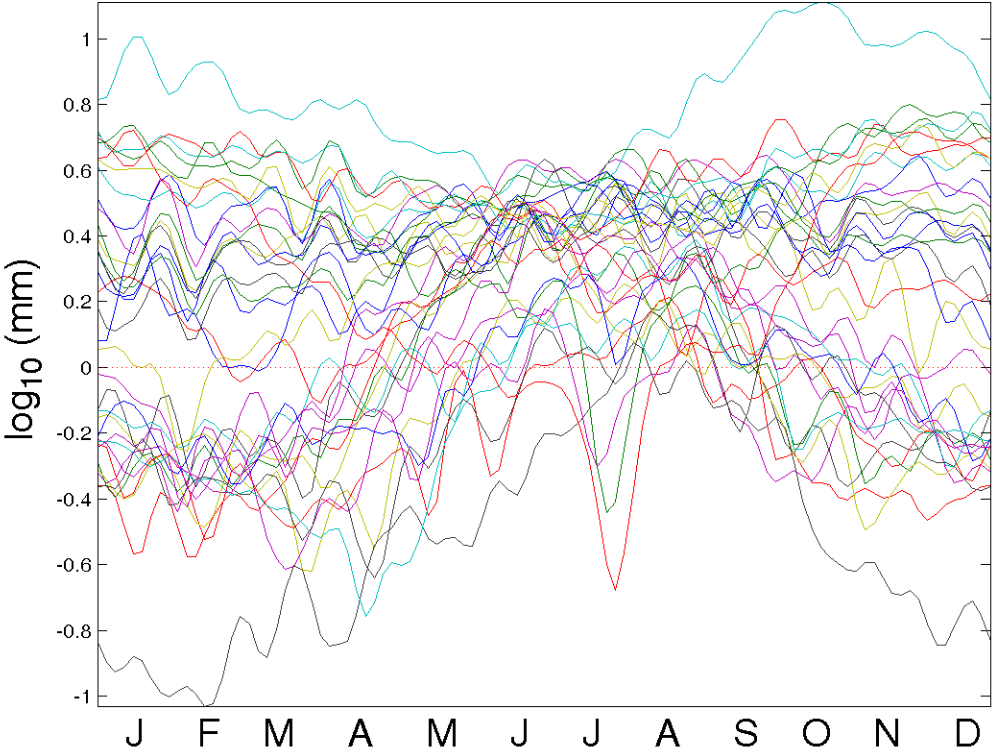
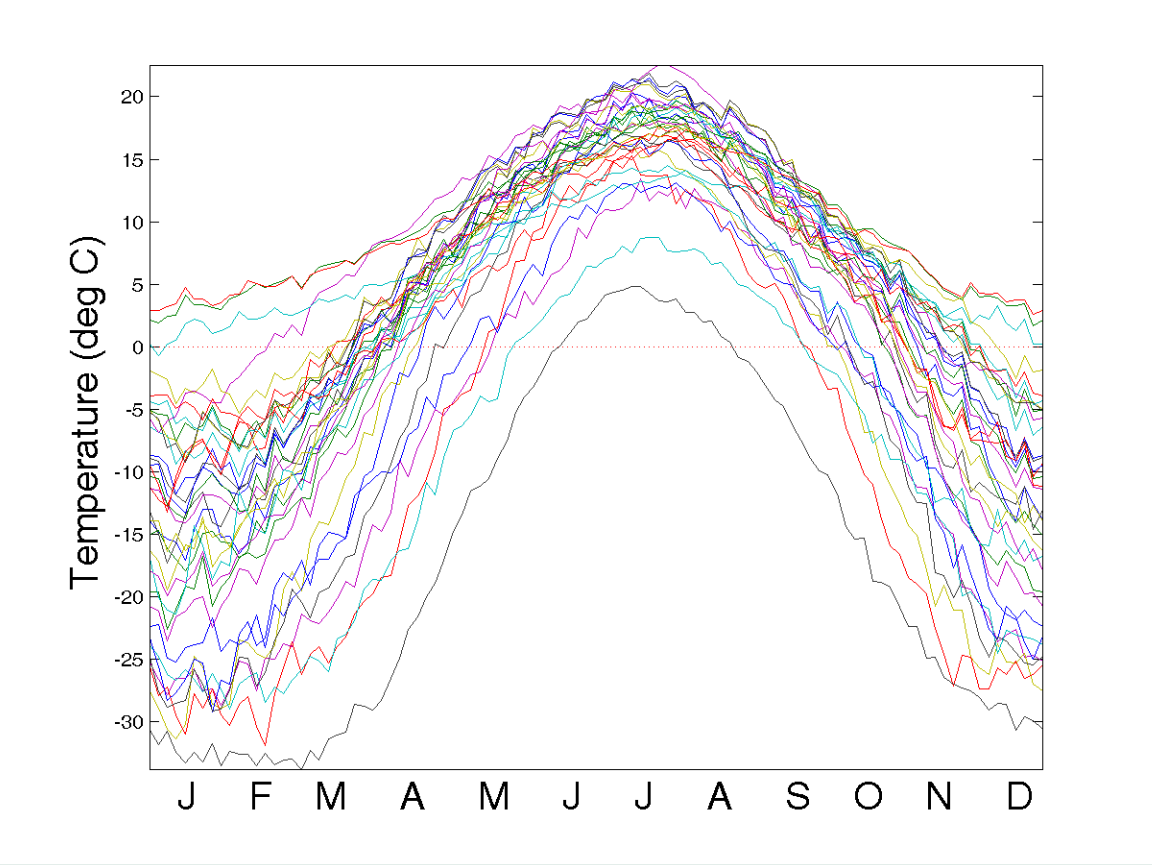
$$\texttt{LogPrec}_i(t) = \alpha(t) + \int_0^{365} \texttt{Temp}_i(s)\beta(s,t)\,ds + \epsilon_i(t)\ .$$

- $\beta(s,t)$ indicates the influence of temperature at time s on precipitation at time t.
- We can use the whole temperature profile because the data are periodic.
- We have already learned from predicting total log precipitation that we will have to apply a roughness penalty to $\beta(s,t)$ as a function of s.
- What about its variation as a function of t?

Log precipitation functions

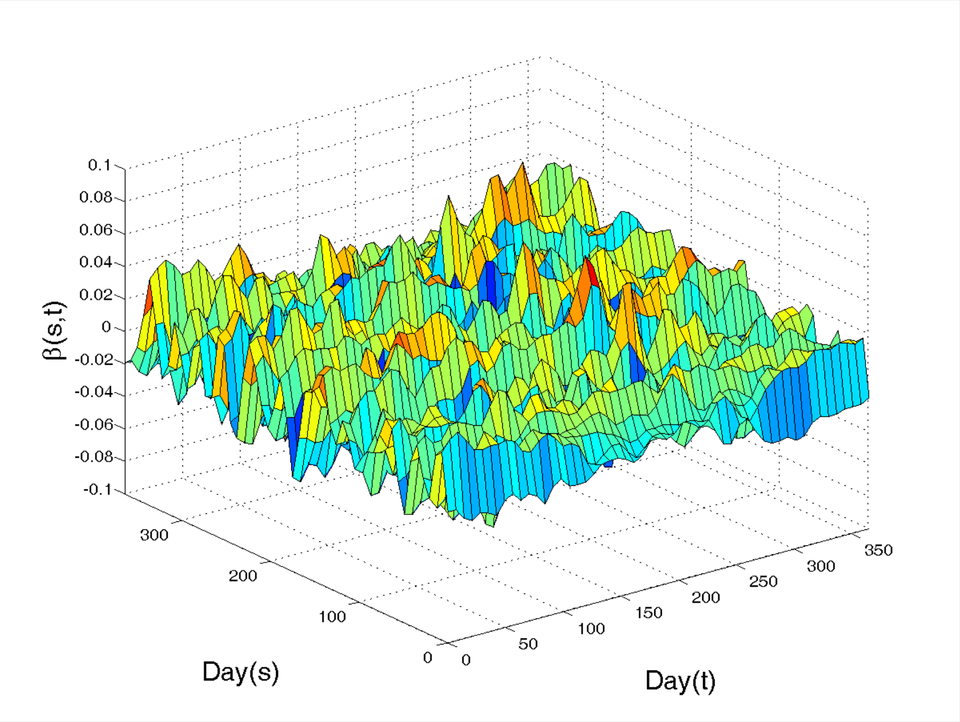


Temperature functions



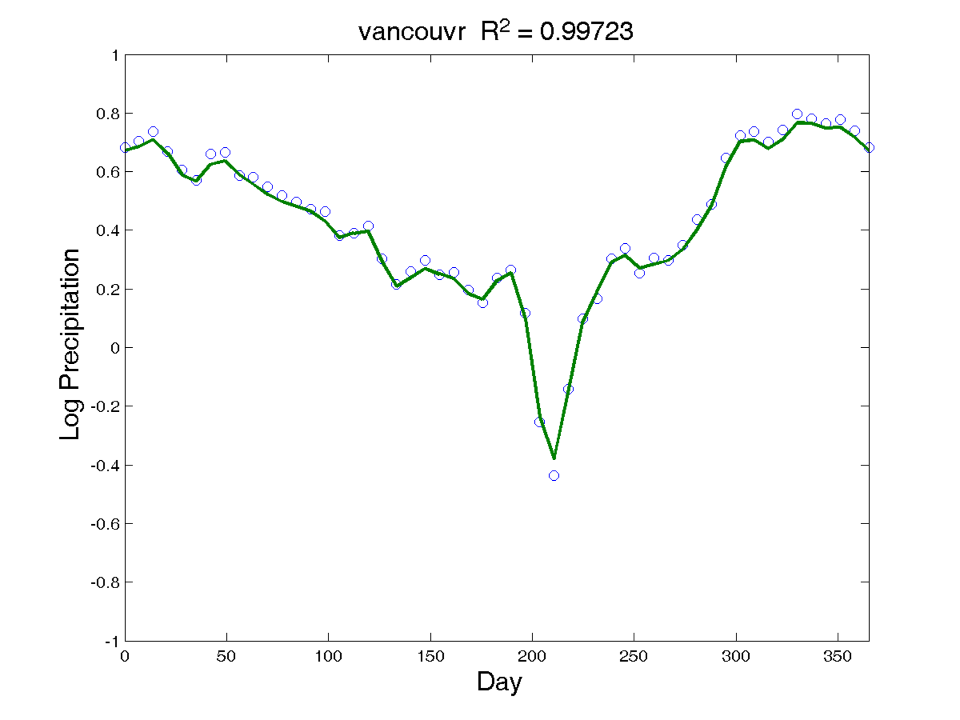
- We apply two harmonic acceleration roughness penalties to $\beta(s,t)$, one for its variation in s, and one for its variation in t.
- Let's see what happens with fairly light penalties on both types of variation.
- We'll look at $\beta(s,t)$ and at the fit to the log precipitation data for Vancouver.

$\beta(s,t)$ has light penalties on s and t



- $\beta(s,t)$ is impossible to interpret.

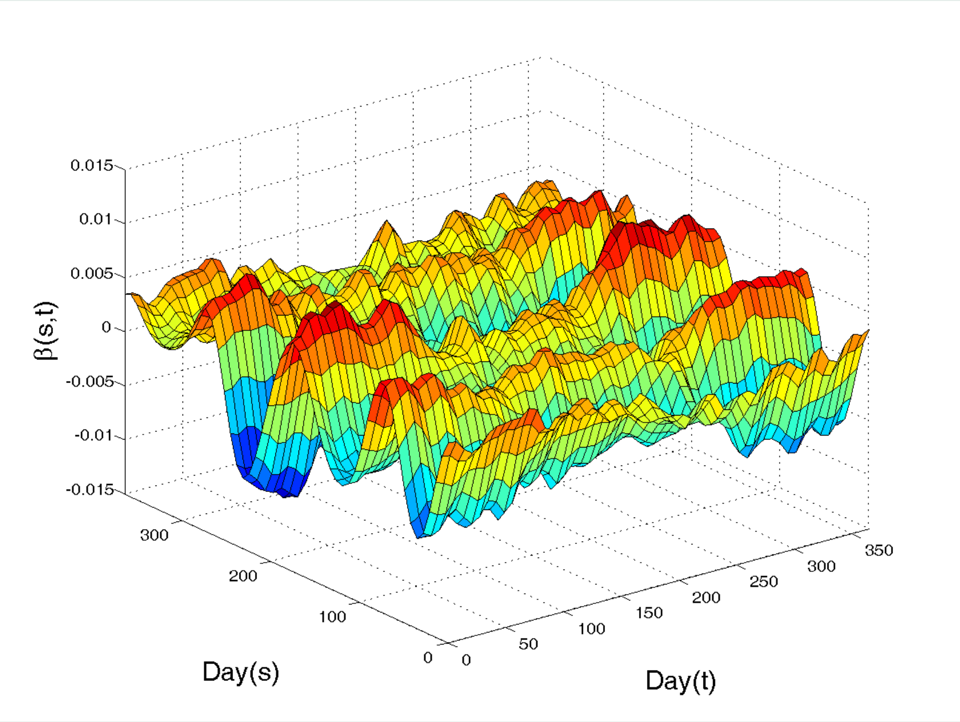
$\beta(s,t)$ has light penalties on s and t



- We seem to have over–fitted Vancouver's data.

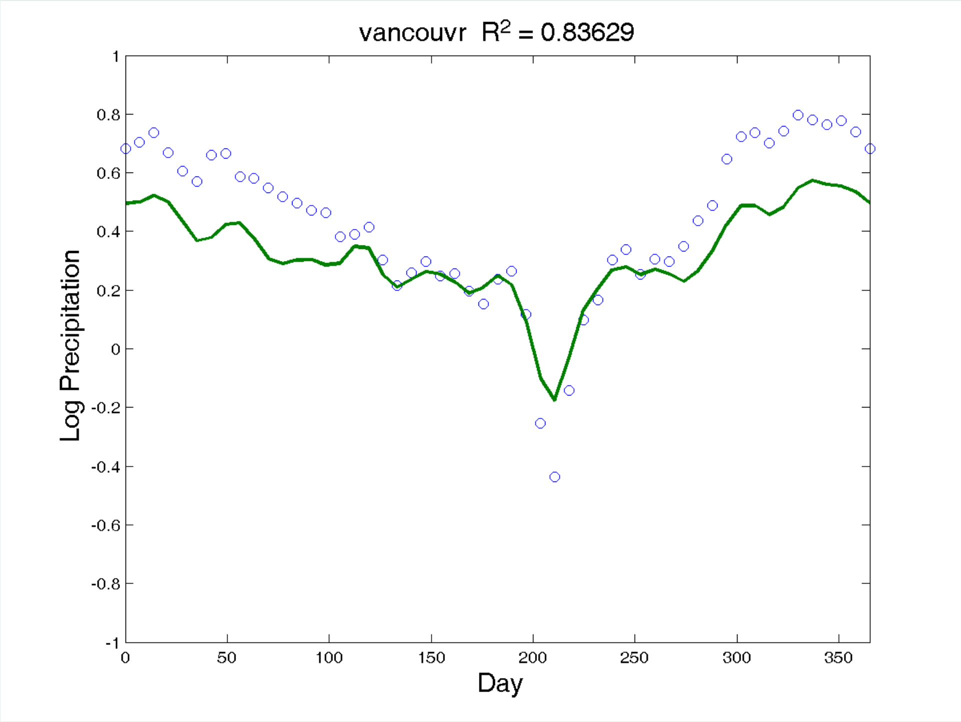
- Let's boost the smoothing parameter for s.

$\beta(s,t)$ has heavy penalty on s and light on t



- $\beta(s,t)$ is interpretable as a function of s but impossible in t.

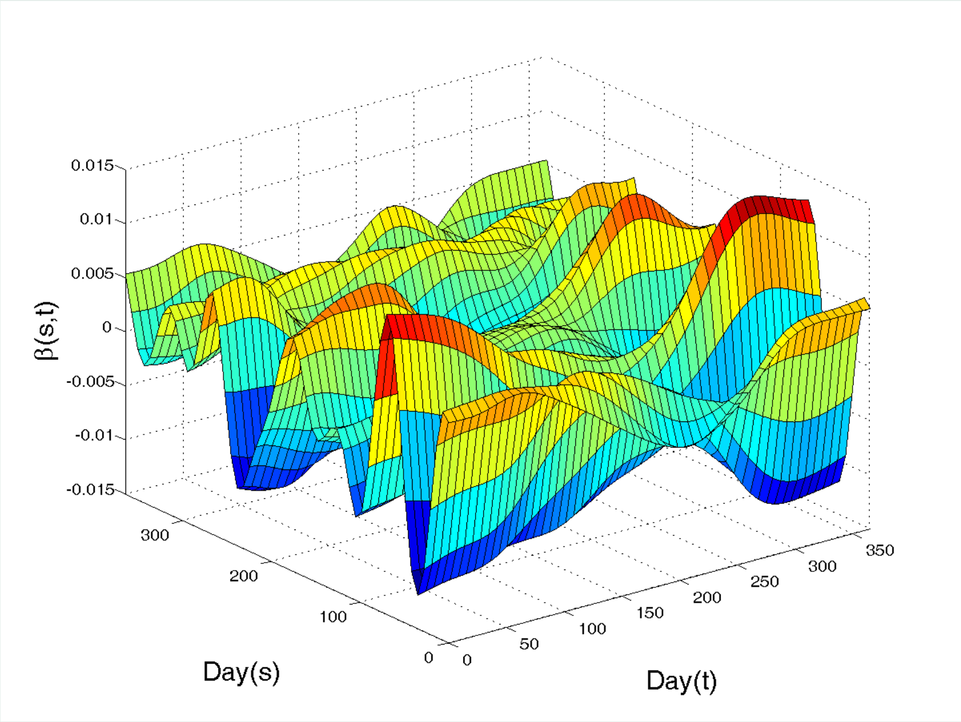
$\beta(s,t)$ has heavy penalty on s and light on t



- We now have a more reasonable fit to Vancouver's data, but the fitting function is too rough.

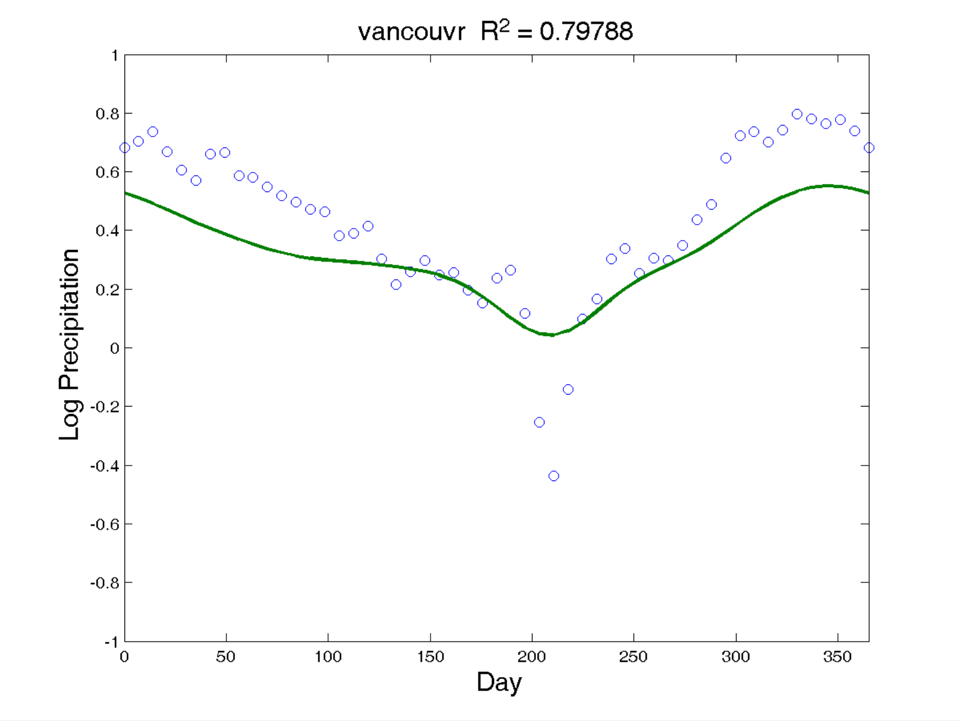
- Let's boost the smoothing parameter for both s and t.

$\beta(s,t)$ has heavy penalties on both s and t



- $\beta(s,t)$ is now smooth in both s and t.

$\beta(s,t)$ has heavy penalties on both s and t



- The fit is reasonable and also smooth.

What we see

- Penalizing the roughness of $\beta(s,t)$ as a function of s prevents over-fitting.
- Penalizing the roughness of $\beta(s,t)$ as a function of t allows us to see how the influence of temperature on precipitation varies from one time to another.
- We can now see that temperature is much more influential in the winter than in the summer.
- The rapid oscillation in s suggests that it is a derivative of temperature that really influences precipitation.

The intercept function $\alpha(t)$