Modelling Change: Sub–second Psychology

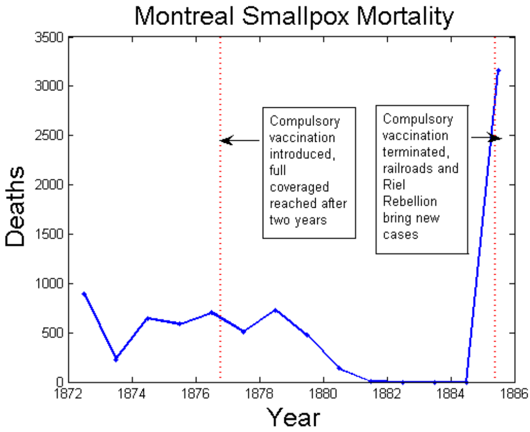
Jim Ramsay
Theo Koulis
Dan Levitin's Lab

- We often collect data on units over time.
- There is an *output measure* $y_i(t)$ that reflects the status of a unit i at time t.
- There are also *input measures* $z_{ij}(t), j = 1, \dots, p$ that indicate the status of various variables thought to affect the output measure.
- We want to study how the status of these units responds to changes in the input variables.
- We especially want to know how a *change* in an input determines the *change* in output.

Examples

- How is driving performance affected by a couple drinks?
- How are golf scores affected by the purchase of a new set of clubs?
- How is pain intensity affected by a dose of morphine?
- How does tumour size respond to radiotherapy?
- How does a couple's social life respond to the birth of a child?
- How does mortality or the incidence of asthma change with an increase in ozone, particulate matter, or other airborne pollutants?

Outline

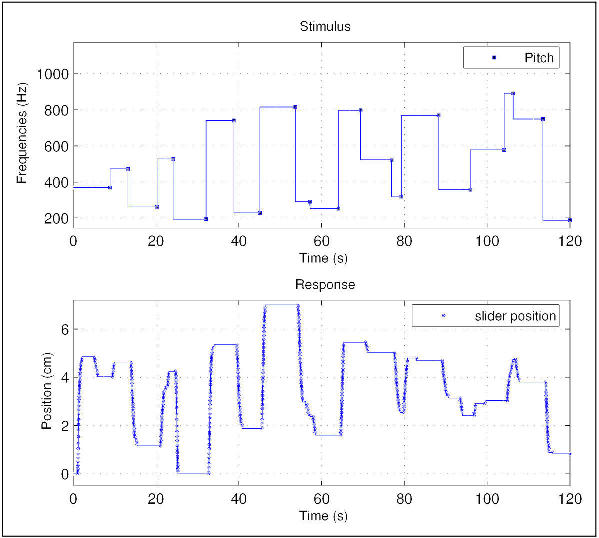


Inputs: Vaccination coverage, infection from outside the city

Outline

- This is an input/output system in music cognition.
- Subjects are asked to follow a series of sequential pitches.
- Subjects adjust a slider on a computer input device (potentiometer).
- If the pitch increases $\longrightarrow$ slider position is increased.
- If the pitch decreases $\longrightarrow$ slider position is decreased.

Input $z(t)$ (top panel) and slider output $y(t)$ (bottom panel)

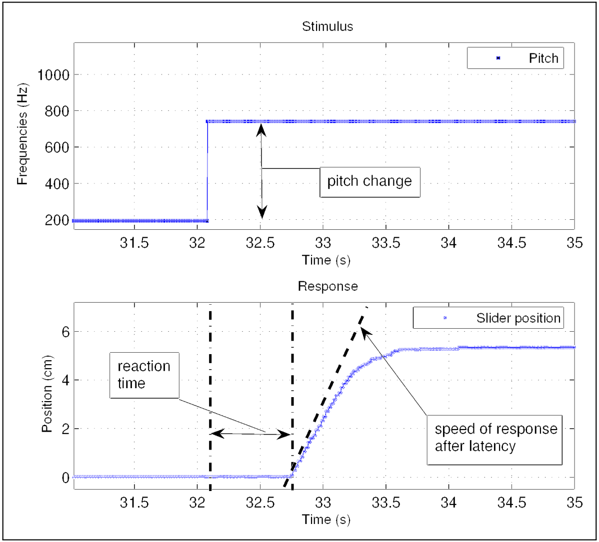


Features of the Slider Data

Psychologists are interested in 3 features of the data.

- Reaction Time: the latency between the onset of a fixed stimulus and the response to it.
- Response Speed: a measure of how fast a subject implements the response to the stimulus.
- Gain: the amount of "energy" required to get to a steady state. It is the ratio of "output to input".

Features: Example from Data



A regression model for change $D\texttt{Slider}(t)$

$$D\texttt{Slider}(t) = -\beta\texttt{Slider}(t) + \alpha\texttt{Pitch}(t-\delta)$$

- $\texttt{Slider}(t)$ is the slider position at time t, with initial condition $Slider(0) = 0$.
- $D\texttt{Slider}(t)$ is the instantaneous slope of the $\texttt{Slider}(t)$ function.
- $\texttt{Pitch}(t)$ is a step function:

$$\texttt{Pitch}(t) = \begin{cases} 0 & \text{if } t < 0 \\ P & \text{if } t \geq 0 \end{cases}$$

- P is the change in pitch.

- The slider position starts at 0 and increases to a limiting value:

$$\texttt{Slider}^* = \frac{\alpha}{\beta} P$$

- The ratio

$$G = \frac{\alpha}{\beta} = \frac{\texttt{Slider}^*}{P}$$

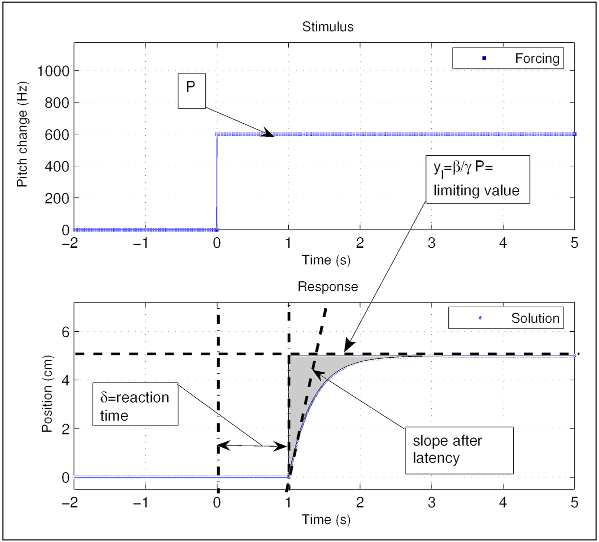
relates the input to the output. We call the ratio G the gain.

- After $\frac{1}{\beta}$ time units, $\texttt{Slider}(t)$ has reached $2/3$ of the final value $\texttt{Slider}^*$.
- After $\frac{2}{\beta}$ time units: $7/8$ of the final value.
- After $\frac{4}{\beta}$ time units: 98% of the final value.
- For this reason, we call the ratio

$$\tau = \frac{1}{\beta}$$

  the response time constant.
- The parameter β is called the response speed.

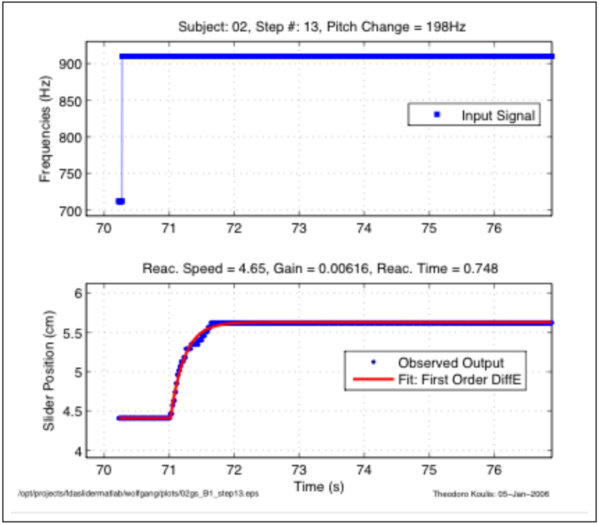
Example: $P = 600$, $\delta = 1$, $\beta = 3$, $\alpha = 0.025$



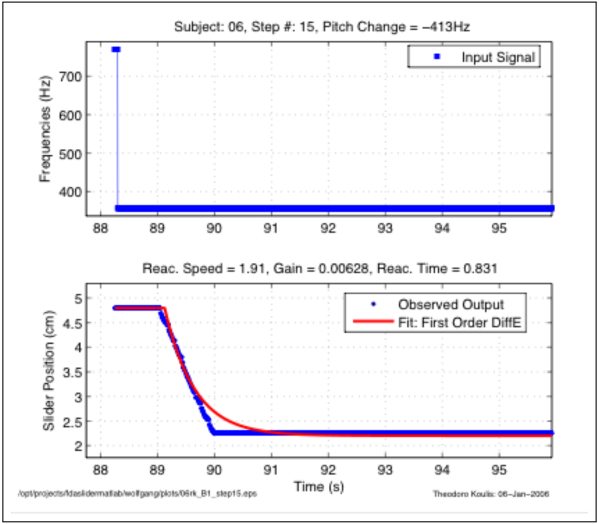
Fitting the Data

- The model does a good job at capturing the shape of the data curves.
- For most cases, the model seems adequate.
- For a few cases, the model does not fit well.
- Even so, we want to keep the simple model to make interpretation easy.

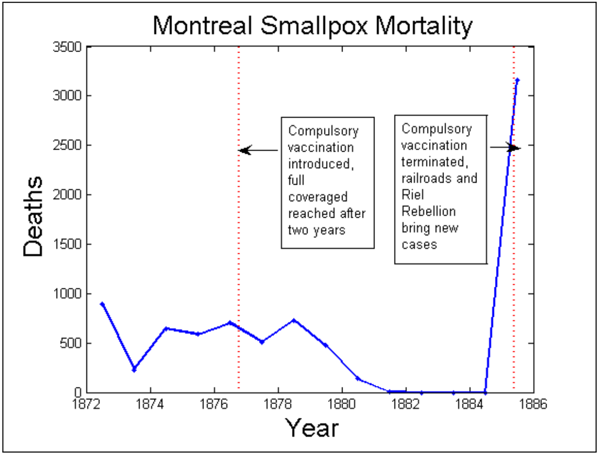
Fitting the Data: Example 1



Fitting the Data: Example 2



Outline



Montreal Smallpox Mortality
Deaths
Year
3500
3000
2500
2000
1500
1000
500
0
1872
1874
1876
1878
1880
1882
1884
1886
Compulsory vaccination introduced, full coveraged reached after two years
Compulsory vaccination terminated, railroads and Riel Rebellion bring new cases

- There is a delay δ of about two years before enough vaccination coverage is reached to be effective.
- Then the disease all but disappears in two years, suggesting a time constant $\tau = 6$ months.
- The epidemic in 1885 goes from just detectable in April to full force in October, suggesting no delay and a time constant of $\tau = 1.5$ months.
- Once the epidemic was obvious to all, full vaccination coverage was almost immediate, and the disease was under control by the end of the year.
- What's most exciting about the smallpox data is the *rate of change* or *dynamics* of the system.

Outline

Consider that there are three basic features of how a system responds to a change in input:

- How quickly does the change take place? ($4/\beta = 4\tau$ time units)
- How much change happens? ($\alpha/\beta = \alpha\tau$ output units per input unit)
- How long before the change begins? (δ time units)

There are other things to model, too, but these are the big three. More exotic characteristics of how the output responds to a change in input might require the use of higher order derivatives, such as $D^2 y(t)$ and etc.